

# Masses of Third Family Vector-like Quarks and Leptons in Yukawa-Unified $E_6$

Aditya Hebbar<sup>a,1</sup>, George K. Leontaris<sup>b,2</sup> and Qaisar Shafi<sup>c,3</sup>

<sup>a</sup>*Department of Physics and Astronomy,  
University of Delaware, Newark, DE 19716, USA*

<sup>b</sup>*Physics Department, Theory Division,  
University of Ioannina, GR-45110 Ioannina, Greece*

<sup>c</sup>*Bartol Research Institute, Department of Physics and Astronomy,  
University of Delaware, Newark, DE 19716, USA*

## Abstract

In supersymmetric  $E_6$  the masses of the third family quarks and charged lepton, t-b- $\tau$ , as well as the masses of the vector-like quarks and leptons, D- $\bar{D}$  and L- $\bar{L}$ , may arise from the coupling  $27_3 \times 27_3 \times 27_H$ , where  $27_3$  and  $27_H$  denote the third family matter and Higgs multiplets respectively. We assume that the SO(10) singlet component in  $27_H$  acquires a TeV scale VEV which spontaneously breaks  $U(1)_\psi$  and provides masses to the vector-like particles in  $27_3$ , while the MSSM doublets in  $27_H$  provide masses to t, b and  $\tau$ . Imposing Yukawa coupling unification  $h_t = h_b = h_\tau = h_D = h_L$  at  $M_{GUT}$  and employing the ATLAS and CMS constraints on the  $Z'_\psi$  boson mass, we estimate the lower bounds on the third family vector-like particles D- $\bar{D}$  and L- $\bar{L}$  masses to be around 5.85 TeV and 2.9 TeV respectively. These bounds apply in the supersymmetric limit.

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<sup>1</sup>E-mail: adityah@udel.edu

<sup>2</sup>E-mail: leonta@uoi.gr

<sup>3</sup>E-mail: shafi@bartol.udel.edu

# Introduction

Yukawa coupling unification (YU)  $h_t = h_b = h_\tau$  of third family charged fermions [1] has received a great deal of attention [2] [3]. While originally implemented in SO(10) grand unification [4], YU has also been studied [5] in the framework of  $SU(4)_c \times SU(2)_L \times SU(2)_R$  (4-2-2) gauge group [6] with some interesting results. For instance, in 4-2-2 models assuming plausible supersymmetry (SUSY) breaking scenarios such as non universal Higgs and scalar masses (NUHM2), one finds that a scenario consistent with the observed (LSP) dark matter abundance and various other experiments requires a NLSP gluino. This is certainly being tested at the LHC. In a different scenario also with t-b- $\tau$  YU but non-universal gaugino masses [7], the colored sparticles lie in the multi-TeV range, while the scalar partners of the leptons are significantly lighter with some also contributing in an essential way to the muon anomalous magnetic moment [8].

In this letter we propose a new extended version of t-b- $\tau$  Yukawa unification inspired by  $E_6$  grand unification [9] [10] [11] [12]. In contrast to SO(10) where t-b- $\tau$  YU only holds in a supersymmetric model, the proposed extended YU, which includes t-b- $\tau$  YU, can also be realized in non-supersymmetric  $E_6$ . In this paper we will restrict our attention to third family extended YU in supersymmetric  $E_6$ . Assuming that the gauge symmetry  $U(1)_\psi$  in  $E_6$  is spontaneously broken in the TeV region, we will exploit the extended YU boundary conditions to infer the lower bounds on the masses of the third family vector-like quarks and lepton doublets predicted in  $E_6$ .

The discussion in the paper proceeds as follows. After a brief overview of t-b- $\tau$  YU, we present some figures based on one-loop RGEs that implement this scenario in the framework of MSSM, as well as MSSM plus three  $(5 + \bar{5})$  multiplets which are present in  $E_6$  models. The one-loop threshold corrections [2] [13] play an important role here. We estimate the unified Yukawa coupling at  $M_{GUT}$  to be approximately 0.6 in MSSM (Figure 1), in agreement with previous work [14]. We then consider MSSM plus three  $(5 + \bar{5})$  multiplets and show a plot displaying gauge coupling unification (Figure 2). We estimate the unified Yukawa coupling at  $M_{GUT}$  in the presence of three  $(5 + \bar{5})$  fields to be 0.35 (Figure 3). To our knowledge, t-b- $\tau$  YU with three additional vector-like families has not been previously discussed. Finally, we discuss extended Yukawa coupling unification motivated by the  $E_6$  invariant Yukawa coupling  $27_3 \times 27_3 \times 27_H$ , where the subscripts denote third family matter fields and the Higgs 27-plet. With the assumption that the MSSM Higgs doublets  $H_u, H_d$  and the SO(10) singlet field  $N$  that breaks  $U(1)_\psi$  arise from this  $27_H$ , we are led to extended YU. Namely, at  $M_{GUT}$ ,  $h_t = h_b = h_\tau = h_D = h_L$ , where  $h_D$  and  $h_L$  respectively refer to the Yukawa couplings to  $N$  of the vector-like color triplets and SU(2) doublets. Employing the lower bound on  $Z'_\psi$  boson mass provided by ATLAS/CMS [15], we estimate the lower bound on

third family D and L masses to be 5.85 TeV and 2.9 TeV respectively. In the  $E_6$  case the Yukawa coupling at  $M_{GUT}$  is estimated to be around 0.3-0.35. In the summary section we note that this value is intriguingly close to the estimates of 0.3-0.5 for the unified third family Yukawa coupling obtained in certain F-theory models based on the  $E_8$  gauge symmetry.

## t-b- $\tau$ Yukawa Unification

In SUSY SO(10) it is plausible that the third family charged fermions in the 16-plet primarily acquire masses from the invariant coupling  $16_3 \times 16_3 \times 10_H$ , with the MSSM doublets contained in  $10_H$ . This yields  $h_t = h_b = h_\tau$  at  $M_{GUT}$  [1]. Note that SUSY plays an essential role here and without it, t-b- $\tau$  YU would not be possible in SO(10) barring additional assumptions. The one loop renormalization group equations for the Yukawa couplings are [16–19]:

$$\begin{aligned}\frac{dh_t}{dt} &= \frac{h_t}{16\pi^2} \left( 6h_t^2 + h_b^2 - \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 \right) \right), \\ \frac{dh_b}{dt} &= \frac{h_b}{16\pi^2} \left( 6h_b^2 + h_t^2 + h_\tau^2 - \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 \right) \right), \\ \frac{dh_\tau}{dt} &= \frac{h_\tau}{16\pi^2} \left( 3h_b^2 + 4h_\tau^2 - \left( 3g_2^2 + \frac{9}{5}g_1^2 \right) \right),\end{aligned}\tag{1}$$

with  $t = \log(Q)$ , where  $Q$  is the renormalization scale.

Using plausible values for the masses of SUSY particles ( $m_{\tilde{t}_1}$ ,  $m_{\tilde{t}_2}$ ,  $m_{\tilde{b}_1}$ ,  $m_{\tilde{b}_2}$  and  $m_{\tilde{g}}$  in the range 2-3 TeV),  $\mu$  parameter ( $\approx 0.5$ -0.7 TeV) and  $A_t \approx 2.5$  TeV, we present two plots (Figure 1) showing how t-b- $\tau$  YU can be realized. Note that in this paper we do not specify any particular SUSY breaking scenario, and these parameters will be employed throughout the paper. The MSSM parameter  $\tan \beta$  is around 50, and we confirm previous results that the unified Yukawa coupling at  $M_{GUT}$  is approximately 0.6 [14].

In the evolution of RGEs we take into account one loop threshold corrections due to sparticle loops [2] [13]. In practice, the largest correction is often to the bottom Yukawa coupling. An approximate expression for the bottom correction (in the limit of the masses of gluino and the top and bottom squarks being approximately equal) is given by:

$$\delta h_b^{\text{finite}} \approx \frac{g_3^2}{12\pi^2} \frac{\mu m_{\tilde{g}} \tan \beta}{m_{\tilde{b}}^2} + \frac{h_t^2}{32\pi^2} \frac{\mu A_t \tan \beta}{m_{\tilde{t}}^2},\tag{2}$$

where  $m_{\tilde{b}} \approx \frac{m_{\tilde{b}_1} + m_{\tilde{b}_2}}{2}$  and  $m_{\tilde{t}} \approx \frac{m_{\tilde{t}_1} + m_{\tilde{t}_2}}{2}$  denote the average bottom and top squark masses respectively.

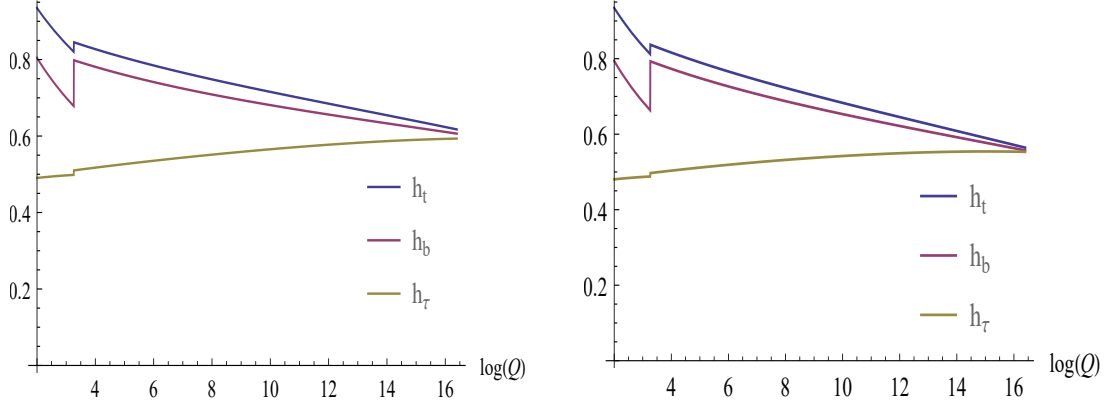


Figure 1: Yukawa coupling unification in MSSM with  $\tan \beta = 50$  (left) and  $\tan \beta = 51$  (right).

## Gauge and t-b- $\tau$ Yukawa unification in MSSM and three $(5 + \bar{5})$ multiplets

We now consider MSSM plus three  $(5 + \bar{5})$  multiplets. The one loop RGEs for the gauge couplings at the 1-loop level are given by:

$$\alpha_i^{-1}(Q) = \alpha_i^{-1}(Q_0) - \frac{b_i}{2\pi} \log \left( \frac{Q}{Q_0} \right). \quad (3)$$

The coefficients  $b_i$  depend on the matter content of the theory. For a SUSY theory with  $n_g$  families,  $n_H$  Higgs doublets and  $n_v$  vector-like families, the coefficients  $b_i$  are given as follows [16–21]:

$$b_1 = 0 + 2n_g + \frac{3}{10}n_H + n_v, \quad (4)$$

$$b_2 = -6 + 2n_g + \frac{1}{2}n_H + n_v, \quad (5)$$

$$b_3 = -9 + 2n_g + n_v. \quad (6)$$

For MSSM and three vector-like  $(5 + \bar{5})$  multiplets, we get the coefficients:

$$\left( \frac{48}{5}, 4, 0 \right) \quad (7)$$

The presence of these new particles increases the value of the unified gauge coupling strength at  $M_{GUT}$  as shown in Figure 2.

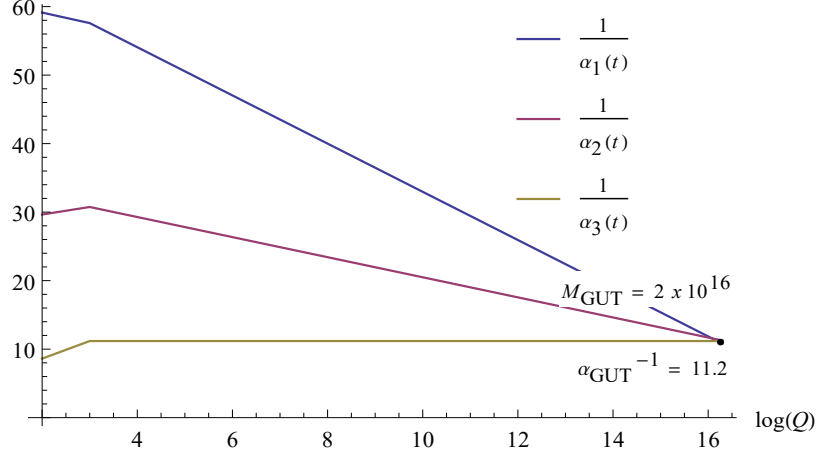


Figure 2: Gauge coupling unification in MSSM and three TeV scale  $(5 + \bar{5})$  multiplets. For the  $E_6$  model, the gauge coupling  $g_\psi$  associated with the extra  $U(1)_\psi$  symmetry is approximately equal to  $g_1$  of the MSSM in between TeV scale and the  $M_{GUT}$  scale.

Next we implement t-b- $\tau$  YU in MSSM plus three families of TeV scale  $(5 + \bar{5})$  particles. The RGEs for the Yukawa couplings remain the same as that for MSSM (1). The evolution of the Yukawa couplings in this case is displayed in Figure 3. As one might expect, the larger gauge couplings have a somewhat greater impact on the Yukawa couplings, and the Yukawa coupling at  $M_{GUT}$  is estimated to be around 0.35.

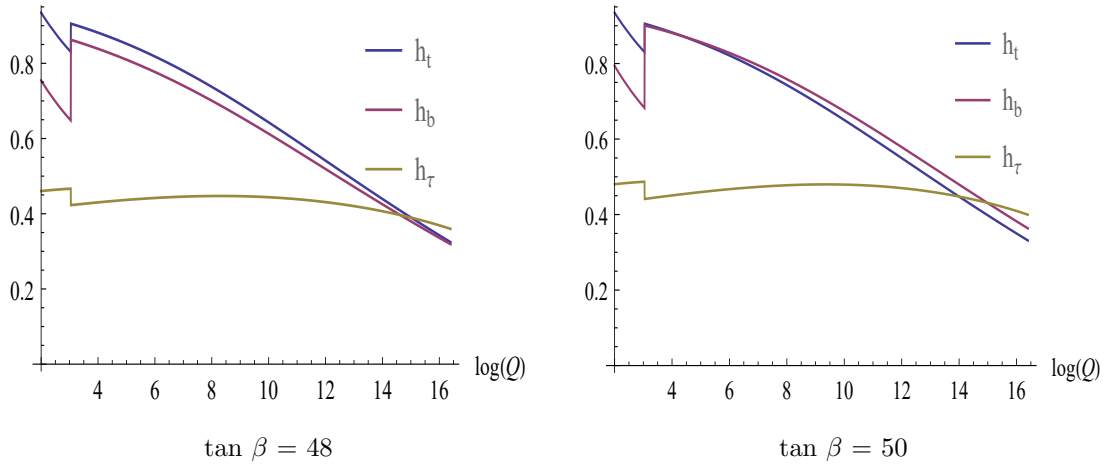


Figure 3: Yukawa coupling unification in MSSM and three TeV scale  $(5 + \bar{5})$  multiplets.

## $E_6$ Yukawa unification

We next proceed to  $E_6$  grand unification and make the simplifying assumption that the Higgs scalars that spontaneously break the MSSM gauge symmetry and  $U(1)_\psi$  arise from the same Higgs 27-plet ( $27_H$ ). Under the decomposition  $E_6 \rightarrow SO(10) \times U(1)_\psi$ , the 27-plet contains the following fields:

$$27 \rightarrow 16_1 + 10_{-2} + 1_4, \quad (8)$$

where the subscripts denote  $2\sqrt{6} Q_\psi$ , with  $Q_\psi$  being the normalized  $U(1)_\psi$  charge [22] [23].

The Yukawa couplings  $D\bar{D}N$  and  $L\bar{L}N$  provide masses to these particles, while the coupling  $H_u H_d N$  can yield the MSSM  $\mu$  term. Indeed, this latter feature provides a good motivation for breaking  $U(1)_\psi$  at the TeV scale [22]. From the third family Yukawa coupling  $27_3 \times 27_3 \times 27_H$ , we obtain the asymptotic YU relation  $h_t = h_b = h_\tau = h_D = h_L$ . The one loop RGEs for Yukawa couplings are given by [16] [22]:

$$\begin{aligned} \frac{dh_t}{dt} &= \frac{h_t}{16\pi^2} \left( 6h_t^2 + h_b^2 - \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{15}g_1^2 + \frac{1}{2}g_\psi^2 \right) \right), \\ \frac{dh_b}{dt} &= \frac{h_b}{16\pi^2} \left( 6h_b^2 + h_t^2 + h_\tau^2 - \left( \frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2 + \frac{1}{2}g_\psi^2 \right) \right), \\ \frac{dh_\tau}{dt} &= \frac{h_\tau}{16\pi^2} \left( 3h_b^2 + 4h_\tau^2 - \left( 3g_2^2 + \frac{9}{5}g_1^2 + \frac{1}{2}g_\psi^2 \right) \right), \\ \frac{dh_L}{dt} &= \frac{h_L}{16\pi^2} \left( 4h_L^2 + 3h_D^2 - \left( 3g_2^2 + \frac{3}{5}g_1^2 + 2g_\psi^2 \right) \right), \\ \frac{dh_D}{dt} &= \frac{h_D}{16\pi^2} \left( 5h_D^2 + 2h_L^2 - \left( \frac{16}{3}g_3^2 + \frac{4}{15}g_1^2 + 2g_\psi^2 \right) \right), \end{aligned} \quad (9)$$

with  $t = \log(Q)$ , where  $Q$  is the renormalization scale.

In the evolution of MSSM gauge couplings we assume that all three vector-like families have masses in the TeV range. However, Yukawa unification as presented above only applies to  $27_3$ , the third family. We assume that there is negligible mixing between the  $\bar{5}$  multiplets contained in the 16-plet and 10-plet of  $SO(10)$ . We also assume that the  $16_H$  component in the  $27_H$  is decoupled from low energy physics, through appropriate fine-tuning.

We explore a few scenarios with varying sparticles masses and  $A$  terms to convince ourselves that the third family unified Yukawa coupling at  $M_{GUT}$  is around 0.35.

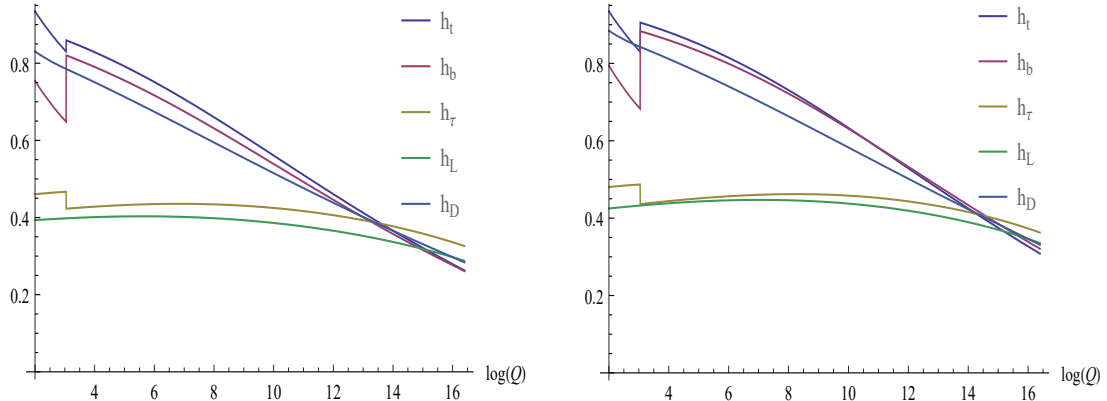


Figure 4: Yukawa coupling unification in  $E_6$  with  $\tan \beta = 48$  (left) and  $\tan \beta = 50$  (right).

Assuming gauge unification, we set the  $U(1)_\psi$  gauge coupling  $g_\psi$  equal to MSSM  $U(1)$  gauge coupling  $g_1$  at  $M_{GUT}$ . With  $b_\psi = \text{Tr } Q_\psi^2 = 9.66$  [22] being approximately the same as  $b_1 = 9.6$  for  $g_1$ , the two couplings stay close to each other between  $M_{GUT}$  and the TeV scale.

From gauge coupling unification (Figure 2) we estimate that the  $U(1)_\psi$  gauge coupling  $g_\psi \approx g_1 = 0.47$  at LHC energies. Together with the lower bound on the  $Z'_\psi$  boson mass of 2.79 TeV [15], the VEV of the  $SO(10)$  singlet scalar field  $N$  is estimated as follows:

$$m_{Z'_\psi} = \frac{4}{2\sqrt{6}} g_\psi \langle N \rangle, \\ \implies \langle N \rangle > 7.3 \text{ TeV}. \quad (10)$$

Combining this with  $h_D$  and  $h_L$  evaluated in the TeV range (Figure 4), we estimate that the lower bounds on the masses of the third family vector-like  $D$  and  $L$  fields are

$$M_D \gtrsim 5.85 \text{ TeV} \quad (11)$$

$$M_L \gtrsim 2.90 \text{ TeV}. \quad (12)$$

Clearly these bounds do not take into account supersymmetry breaking.

## Summary

We have extended the idea of  $t$ - $b$ - $\tau$  Yukawa unification in supersymmetric  $SO(10)$  by including the Yukawa couplings of vector-like fields that appear in the matter 27-plet of  $E_6$ . Requiring unification of these Yukawa couplings at  $M_{GUT}$ , and taking into account both gauge coupling unification and the ATLAS/CMS lower bound on

the appropriate  $Z'$  boson mass enables us to estimate lower bounds on the third family color triplet and  $SU(2)$  doublet vector-like fields. We should remark here that the first two families of vector-like quarks and leptons may well be considerably lighter than the third family, in which case some of them may be accessible at the LHC. It is interesting to note that certain F-theory constructions utilizing  $E_8$  and  $E_6$  gauge symmetries predict unified Yukawa coupling  $\sim 0.3$ - $0.5$  for the third family fields including the vector-like ones contained in  $E_6$  [24]. This is intriguingly close to the value of  $0.3$ - $0.35$  that we estimate for extended YU in the  $E_6$  model and deserves further exploration.

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